Propositional Calculus

Boolean Algebra

Statement:

Def:- A statement is a sentence which is either true or false.

for example- ① I am a student of M.C. College"
② Barpeta is state of India.

These two are statement. The truth value of statement ① is 'true' and of ② is false.

Again ③ what is your name.
④ Let us go to college. - are not statemen

If a statement's truth value is True it is denoted by 'T' and false is denoted by 'F'

Statements are usualy denoted by small english alfabets.

eg q = This is a red ball.
p = There is a dog.

Operations on statements:-

① Conjunction: It is a binary operation. If we join two statement by the word "or" then it is called conjunction of two statements. it is denoted by '$\vee$'

eg $p \vee q \Leftrightarrow p$ or q

$p \vee q$ is true iff any one of p or q is true.

③ Disjunction:- It is also a binary operation. If two statements p, q are joint by the word 'and' then it is called disjunction of the statement and it is denoted by $\wedge$.

eg. $p \wedge q \Leftrightarrow p$ and q.

$p \wedge q$ is true iff both p and q are true.

④ Negation:- It is an unary operation. If p is a statement then the negation of p is denoted by '$\sim p$' is read as 'not p'.

$\sim p$ is true iff p is false

⑤ Truth Table:- If we denote all the possible truth values of a statement or combination of statements in a table then this table is called truth table:-

p	q	$\sim p$	$\sim q$	$p \vee q$	$p \wedge q$	$(\sim p) \vee q$	$p \vee (\sim q)$	$(\sim p) \vee (\sim q)$	$(\sim p) \wedge (\sim q)$
T	T	F	F	T	T	T	T	F	F
T	F	F	T	T	F	F	T	T	F
F	T	T	F	T	F	T	F	T	F
F	F	T	T	F	F	T	T	T	T

⑤ There is another binary operation called 'Exclusive Or'. If p and q are two statements then the exclusive or denoted by $p \oplus q$ is defined as "either p or q, but not both".

The truth table of exclusive or is

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

6. Implication:- If p and q be two statement. Then the statement "if p then q" is called implication and it is denoted by '$p \rightarrow q$'. This statement is false if p is true and q is false. The truth table is -

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

7. Biconditional: Two statements p and q are also can join by 'p if and only if q'. This statement is called biconditional statement, and it is denoted by '$p \leftrightarrow q$'

Truth table -

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

This statement is false if one is false and other is true.

8. Logicaly equivalent or Equivalent Statements:-
Two statements are equivalent if both have identical truth table.

Q:- ① Construct a truth table of $(\sim p \vee q) \wedge (q \to p)$

p	q	$\sim p$	$\sim p \vee q$	$q \to p$	$(\sim p \vee q) \wedge (q \to p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

② Construct truth table for the following statements
(i) $(\sim p \vee q) \wedge (p \wedge \sim q)$
(ii) $(\sim p \wedge q) \vee (p \wedge \sim q)$
(iii) $(\sim p \wedge \sim q) \vee (p \wedge q)$

9. Tautology:- If all the entries of a statement in a truth table are 'T', then the statement is called a Tautology.
For example - $(p \vee \sim p)$ is a Tautology.

10. Contradiction:- If all the entries of a statement in a truth table are "F" then it is called a contradiction.
For example $(p \leftrightarrow \sim p)$ is a contradiction.